Turbulent Wall-Jets in Conical Diffusers

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Mean flow measurements in wall-jets in conical diffusers subjected to annular injection at the inlet are reported. The effects of diffuser angle and the ratio of the injection to inlet centerline velocity have been studied. The experiments indicate that for small values of a suitably defined pressure gradient parameter β, the velocity profile follows a universal logarithmic law in the fully turbulent region close to the wall. It has also been found that, at small values of β, the development of wall-jets in conical diffusers can be correlated with that of plane wall-jets in zero pressure gradient by using suitably stretched parameters. The restriction on the magnitude of the pressure gradient parameter is more severe for the successful correlation of the parameters of the jetlike outer layer than for those of the boundary-layerlike inner layer. The wall-jet development in conical diffusers has also been computed by solving the relevant partial differential equations using a two-layer mixing length model of turbulence, with two adjustable constants. It is found that this method yields satisfactory prediction of the wall-jet development.

Nomenclature

C_f = drag doefficient $\tau_w/\frac{1}{2}\rho U_e^2$
l_m = Prandtl mixing length
P = static pressure
R = radius of the diffuser at any station
R_c = radius of the diffuser at the slot tip
r = distance from the axis of symmetry
t_c = slot width
U = x-component of mean velocity
U_j = injection velocity
U_e = freestream velocity at any station
U_o = velocity in the inlet pipe
U_m = maximum velocity in the wall-jet (see Fig. 2)
U_{min} = minimum velocity in the wall-jet (see Fig. 2)
U^+ = nondimensional velocity U/u_*
u_* = friction velocity $(\tau_w/\rho)^{1/2}$
u = x-component of turbulent fluctuation velocity
v = y-component of turbulent fluctuation velocity
V = y-component of mean velocity
x = distance from the slot tip measured along the diffuser wall
x_e = distance along the wall measured from the virtual origin
x_o = position of virtual origin
y = normal distance from the diffuser wall in the inward direction
y^+ = nondimensional distance from the wall yu_*/ν
$y_{1/2}$ = half width of the jet (see Fig. 2)
y_m = distance from the wall to the point of maximum velocity (see Fig. 2)
y_1 = characteristic width of the wall-jet [Eq. (14)]
β = pressure gradient parameter defined in Eq. (2)
θ = half angle of the diffuser
δ = characteristic width of type IV velocity profile (see Fig. 2)
κ, λ = Prandtl mixing length constants
ν = laminar viscosity
ρ = density
τ = laminar shear stress
τ_w = shear stress at the wall

1. Introduction

THE study of turbulent wall-jets has been of considerable interest to many investigators because of their application in film cooling, boundary-layer control on aerofoils, etc. The study in the present case, however, was motivated from a different application. It was found by the author[1,2] that pressure recovery obtained in a wide-angle conical diffuser could be very significantly improved by injecting a stream of fluid at a high velocity tangentially through an annular slot situated at the inlet to the diffuser. The flow adjacent to the diffuser wall, in such a case, can be considered to be an axisymmetric, confined wall-jet in a moving stream subjected to an adverse pressure gradient. In order to predict the performance of the diffuser in such a case, it is necessary to understand the behavior of this wall-jet under various operating conditions.

A number of investigations are reported in literature concerning wall-jets in a moving stream subjected to zero pressure gradient.[3-12] Some of these investigators have also reported data on wall-jets in adverse pressure gradients. All these data except those of McGahan refer to two-dimensional plane flows. McGahan's[6] data were obtained on a cylindrical rod placed longitudinally in the stream. The rod was of constant diameter the pressure gradient being created by diffusing the fluid through a porous outer cylinder. The flow in this case was thus essentially an external boundary layer with only possible transverse curvature effects but no other effects due to the axisymmetric configuration. The author is not aware of any data on wall-jets confined in a duct and subjected to adverse pressure gradients.

With regard to the calculation of wall-jet flows analytically the most common practice seems to be to use some kind of an integral method and assume similarity to exist over different regions of the wall-jet. The exception is in the case of Kacker and Whitelaw[9] who solved the partial differential equations of the walljet assuming Prandtl's mixing length hypothesis.

The investigation reported now had, as its objectives, the following: (1) an experimental study of the development of wall-jets in conical diffusers and (2) development of an analytical procedure for calculating the development of such flows.

2. The Experimental Investigation

2.1 The Experimental Set-Up

The experimental apparatus used in the present investigation is the same as that used for the study of the performance of conical diffusers with inlet injection and is described in detail in Ref. 1. A schematic diagram of the set-up is shown in Fig. 1. The conical diffusers in which the wall-jet flows were obtained were moulded out of fiber glass and had a nominal inlet diameter of 12 in. and area ratio of 3. Three diffusers were used and these had half-cone angles of 5°, 10°, and 15°. The annular slot through which the jet of air was injected was 0.25 in. in width and adjusted to be uniform around the circumference

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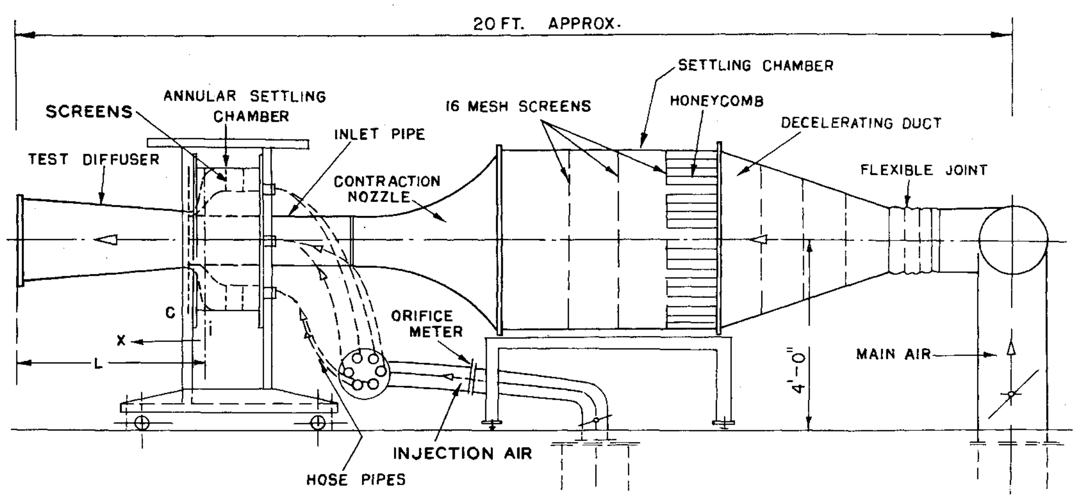

Fig. 1 Layout of the test rig.

and coaxial with the diffuser. The slot exit was so shaped as to direct the jet parallel to the diffuser wall. The velocity profile across the slot exit was flat over 90% of the slot width. Typical velocity distributions across the diffuser are sketched and pertinent nomenclature explained in Fig. 2.

The velocity in the inlet pipe was maintained at about 70 fps. The injection velocities were adjusted such that separation did not occur over any part of the diffuser. The injection velocities ranged from 85 to 150 fps. Velocity profiles and wall shear stress were measured at several stations along the diffuser during each run. The stations at which velocity traverses were made are shown in Fig. 3. Figure 4 gives the distribution of the centerline velocity along the diffuser for the different runs. The legend in Fig. 4 gives the particulars of the different runs. The velocity traverse at each station was made in a direction perpendicular to the diffuser wall. Over the range of the experiments reported here, the effect of misalignment between the total head tube and the flow direction is considered to be negligible. Total head was measured by using a round total head tube of 0.028 in. o.d. Wall shear stress was measured with a Preston tube using Patel's[13] calibration curve. Care was taken to see that the size of the Preston tube used was sufficiently small so that its effective opening lay within the viscous sublayer or buffer layer. It is expected that the behavior of the flow within the buffer layer (say $y^+ < 10$) will not be different from that in the corresponding region of fully developed pipe flow, in which the calibration was made by Patel. Data on plane wall-jets from Ref. 4 appears to justify this expectation. Also, wall shear stress values were obtained independently in one case (viz., run 500) by extrapolation, to the wall, of hot wire data on shear stress distribution across the wall-jet. The hot wire data agreed within 10% with the Preston tube data as can be seen from Fig. 10. It is, therefore, believed that the present wall-shear stress data are of acceptable accuracy.

2.2 Experimental Results and Discussion

2.2.1 The wall-layer

Data of Kruka and Eskinazi[4] on plane wall-jets in zero pressure gradient have shown that the velocity profile in the fully turbulent part of the wall region exhibits a universal logarithmic distribution of the form

$$U^+ = A \log y^+ + B \tag{1}$$

The constant A and B are 4.83 and 11.5, respectively. The value of B is thus significantly different from the value 5.5 for boundary layer on a flat plate. In the case of wall-jets in conical diffusers the half cone angle θ and the pressure gradient dP/dx can also be expected to influence the velocity profiles in the wall region. Figure 5 shows a few typical velocity profiles in the wall region. It can be seen that data corresponding to the

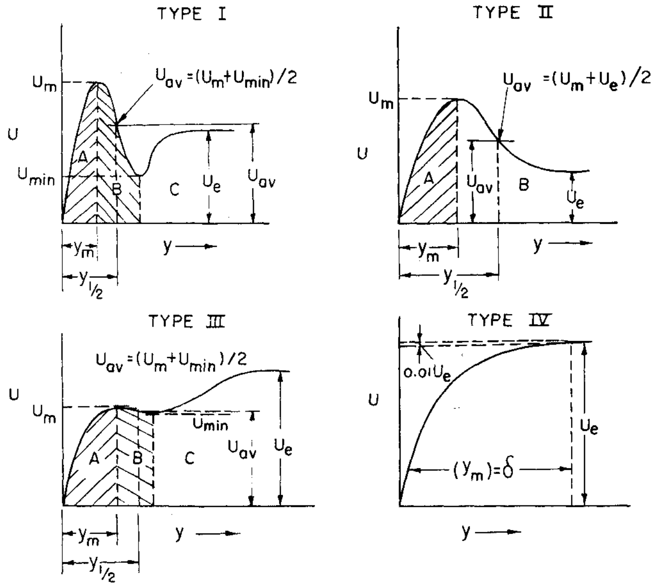

Fig. 2 Types of velocity profile observed and relevant nomenclature.

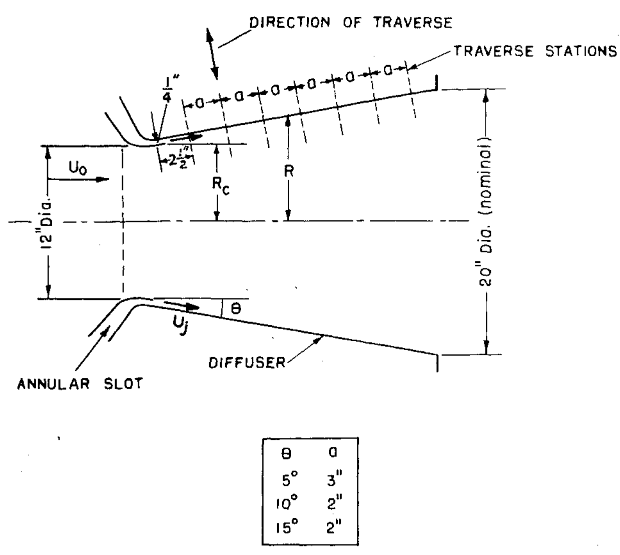

Fig. 3 Details of stations at which velocity traverses were made.

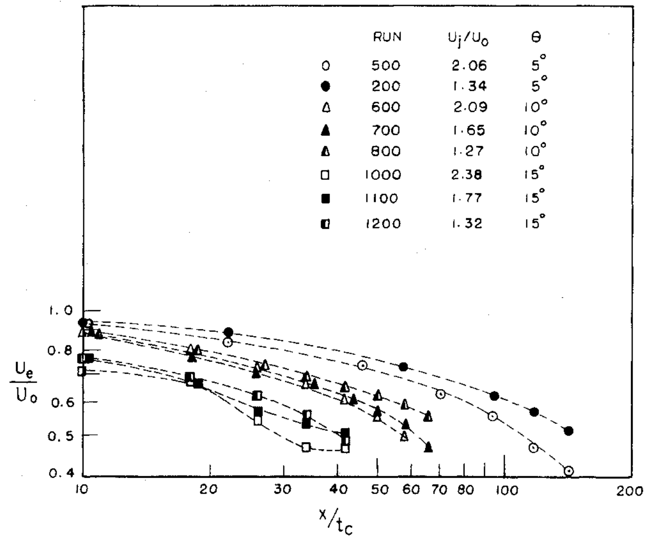

Fig. 4 Distribution of the centerline velocity along the diffuser.

traverses listed in the top box in Fig. 5 exhibit a nearly universal semi-logarithmic distribution. It is clear from the details given in the top box in Fig. 5 that these data correspond to different values of U_j/U_o, θ, and x/t_c. The only condition common to all of them is that the pressure gradient expressed in terms of a Clauser-type pressure gradient parameter β is "small," where β is defined as

$$\beta = (y_m/\tau_w)\, dP/dx \tag{2}$$

The value of β is seen to be less than or equal to 1 for all the preceding cases. A least square fit to the data indicates that, for $\beta \gtrsim 1$, the velocity distribution can be represented by the equation

$$U^+ = 5.5 \log y^+ + 11.2 \tag{3}$$

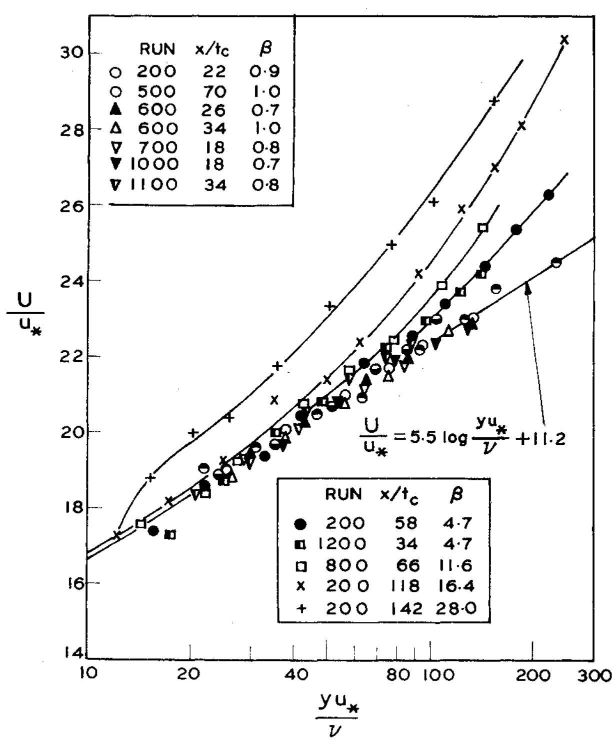

Fig. 5 Velocity profiles in the wall region.

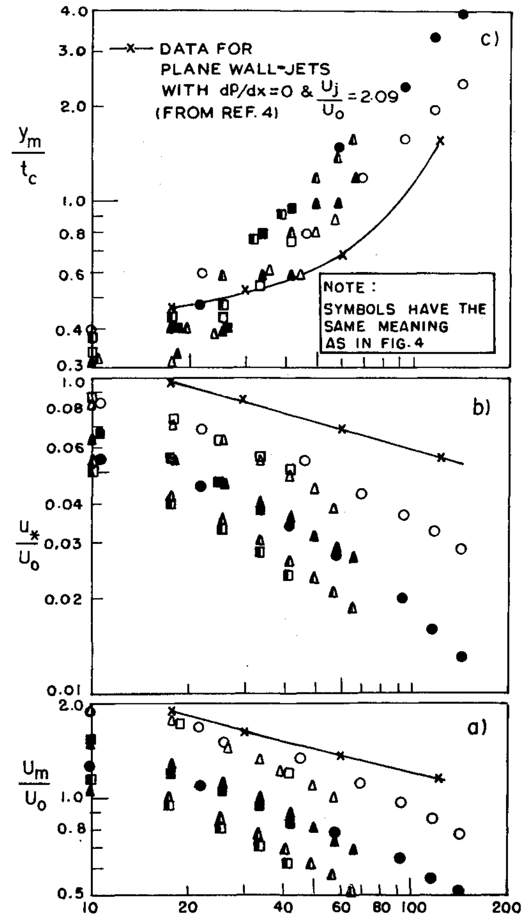

Fig. 6 Development of U_m, u_*, and y_m.

This can be compared with the log law for plane wall-jets in zero pressure gradient.

As the pressure gradient parameter β increases the velocity profiles start deviating from the logarithmic distribution. The distributions for some values of β are shown in Fig. 5. The details of the experimental conditions are given in the bottom box in the figure. It can be seen that the two profiles corresponding to $\beta = 4.7$ are very nearly identical. In fact, the same behavior was observed at lower values of β also ($1 < \beta < 4.7$), i.e., the velocity profiles for a given value of β were independent of θ, U_j/U_o and x/t_c. These profiles are not shown in Fig. 5 as they would crowd the figure with too many points. Unfortunately, however, it was not possible to find, for $\beta > 4.7$, two or more profiles with the same value of β. However, a study of the velocity profiles in Fig. 5 for large values of β shows that the deviation from the logarithmic profile systematically increases with the increase in β. Thus, β appears to be an important parameter characterizing the velocity profile in the wall-region. Although data for $\beta < 4.7$ suggest that the velocity distribution in the wall region is universal for a given value of β, more data are needed for large values of β to arrive at a definite conclusion.

2.2.2 The development of the wall-jet

If the wall-jet profiles attain a state of self-preservation, the only two important parameters characterizing the wall-jet development are the maximum velocity U_m and the half width $y_{1/2}$. However, as can be seen from Fig. 2, such a state is not attained in the present case and one needs more parameters to define the wall-jet status. It would be interesting, therefore, to examine the development of some of these parameters. Figure 6 shows the development of the parameters U_m, u_*, and y_m while

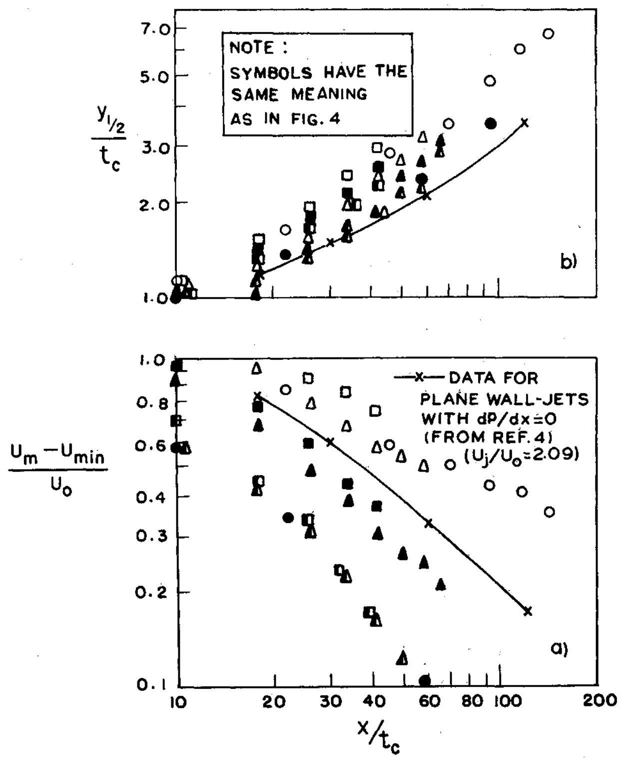

Fig. 7 Development of $(U_{max}-U_{min})$ and $y_{1/2}$.

Fig. 7 shows the development of (U_m-U_{min}) and $y_{1/2}$. These were the only parameters that remained well-defined over most of the flow and could, therefore, be measured with reasonable definiteness. However, even among these, the parameters in Fig. 7, became ill-defined in some runs towards the end of the diffuser and such data are not shown in the figure. The data for plane wall-jet for $U_j/U_o = 2.09$ from Ref. 4 are also shown in these figures. It can be seen that the behavior of the present wall-jets is different from that of the plane wall-jets, as indeed, one would intuitively expect. For instance, the values of U_j/U_o for runs 500 and 600 are nearly the same as for the plane wall-jet. But the development of the parameters is different in each case. A careful study of Figs. 6 and 7 will also reveal that it is difficult to identify the individual effects of U_j/U_o and θ on the flow development. Although most of the data in Fig. 6 indicate that the dimensionless velocity scales U_m/U_o and u_*/U_o depend only on U_j/U_o and not on θ, the data from run 200 show a different trend. Similarly, although some of the data in Fig. 6c indicate that y_m/t_c depends only on U_j/U_o, others (e.g., data from run 800) suggest dependence on θ also. From Fig. 7a, it is seen that $(U_m-U_{min})/U_o$ depends only on U_j/U_o and also that the rate of decay of the velocity scale increases with the decrease in U_j/U_o. Figure 7b suggests that $y_{1/2}/t_c$ depends both on U_j/U_o and θ.

In order to bring about some measure of order among the present data and to compare with the plane wall-jet data, it is necessary to introduce suitably stretched variables for describing the wall-jet flow. We shall first consider parameters characterizing the A-layer in Fig. 2.

Plane wall-jet study in Ref. 4 has shown that y_m grows linearly with x/t_c and the growth, for all values of U_j/U_o, can be expressed by the universal relation

$$y_m/t_c = 0.0109(x+x_o)/t_c \tag{4}$$

where x_o is the distance of the virtual origin from the slot tip. Similarly, it has also been found in Ref. 4 that U_m decays according to the relation

$$U_m \propto \{(x+x_o)/t_c\}^{-\alpha} \tag{5}$$

The exponent α varies with U_j/U_o. However, it varies by less than 10% in the range $1 < U_j/U_o < 2.5$ which is the range of interest to the diffuser designer. At $U_j/U_o = 2.09$, its value is 0.26.

In conical diffusers, the axisymmetric diverging shape of the duct can be assumed to introduce two types of effects on the flow: 1) a geometrical effect due to the increase of radius with distance from the slot tip. This effect can be represented by the quantity R/R_c. One can see intuitively that even if turbulent mixing between the jet and the surrounding fluid was not present, the jet velocity would still decay and jet width would still increase because of the diffuser geometry. 2) The pressure gradient effect, due to the presence of an adverse pressure gradient in the diffuser. Unfortunately, this effect is more difficult to be accounted for. If the wall-jet were in equilibrium throughout, then a pressure gradient parameter such as β can represent completely the effect of the pressure gradient. Otherwise, the precise history of the flow becomes important in addition to the local value of β. Since the flow is confined in the duct, the history of the flow is decided by the interaction of the wall-jet and the inviscid core-flow. Methods for accounting for this—for example, by solving the governing equations—presently do not exist.

If we consider only flows for which β is small, we can expect U_m/U_j and y_m/t_c to be functions of $(x+x_o)/t_c$ and R/R_c only. Guided by intuition that U_m decreases and y_m increases because of the effect of R/R_c, and choosing a simple functional relationship, we shall write, for small values of β,

$$(U_m/U_j)\cdot R/R_c = f_1(x_e/t_c) \tag{6}$$

and

$$(y_m/t_c)\cdot R_c/R = f_2(x_e/t_c) \tag{7}$$

where $x_e = (x+x_o)$ and f_1 and f_2 are universal functions.

Figure 8 shows the data plotted in terms of the variables of Eqs. (6) and (7). It can be seen from Fig. 8a, that with the exception of the data closest to the slot, the rest of the data for y_m, except in the case of run 200, exhibit a universal behavior. The average value of β in all the runs shown in Fig. 8a (except

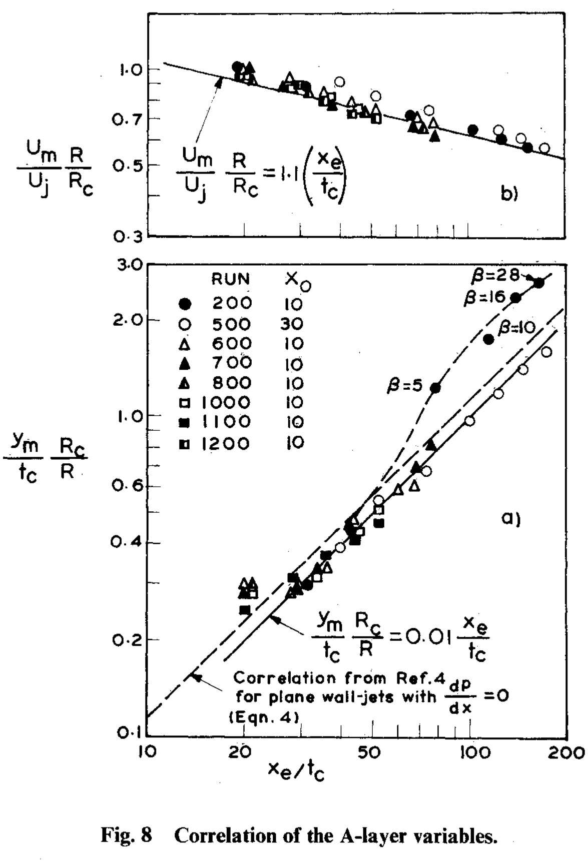

Fig. 8 Correlation of the A-layer variables.

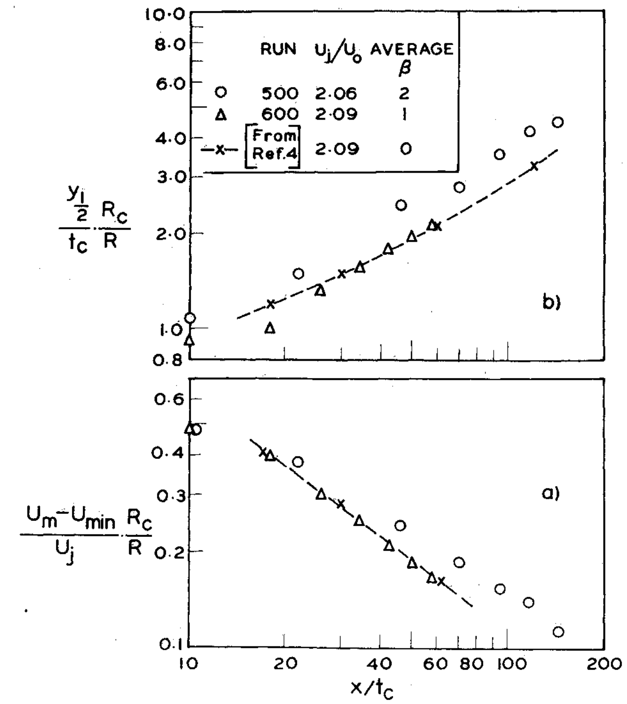

Fig. 9 Correlation of the B-layer variables.

run 200) was less than 4. The variation of y_m along the diffuser for these runs can be closely represented by the straight line

$$(y_m/t_c) \cdot R_c/R = 0.01(x_e/t_c) \tag{8}$$

The corresponding universal line [Eq. (4)] for plane wall-jets with $\beta = 0$ is also shown in Fig. 8a.

The average value of β was greater than 4 for runs 200, 800, and 1200. The data for run 200 only is shown in Fig. 8a to illustrate the effect of pressure gradient. The value of β is indicated for each data point. The deviation from the universal line can be seen to be significant and the precise amount of deviation can be predicted as already stated only by methods presently not available such as solution of the governing equations.

The decay of velocity maximum is shown in Fig. 8b. A universal behavior is observed for all data including (surprisingly) even those for runs 200, 800, and 1200. This is possibly because the rate of decay is mildly dependent on U_j/U_o. Smaller values of U_j/U_o tend to slow down the rate of decay whereas the larger values of β associated with these low injection rates tend to increase the decay rate. It appears that over the range of injection rates studied, the opposing trends balance each other. The full line in Fig. 8b corresponds to plane wall-jets ($R = R_c = \infty$) with $U_j/U_o = 2.09$ and is represented by the equation

$$(U_m/U_j) \cdot R/R_c = 1.1\,(x_e/t_c)^{-0.26} \tag{9}$$

It is seen that most of the present data correlate reasonably well with the plane wall-jet data. The behavior of u_* needs no separate discussion since it was found to be generally similar to that of U_m. This is evident from Figs. 6a and 6b. The expression that was found to describe best the variation of u_* is

$$(u_*/U_j) \cdot R/R_c = 0.15(x_e/t_c)^{-0.4} \tag{10}$$

Departure from this equation was observed only at values of β greater than 10, such as at the last three stations for run 200. (See Fig. 6b).

We shall now consider the development of the variables characterizing the B-layer in Fig. 2. The problem now is more complex because of two reasons. 1) The B-layer is very sensitive to the upstream history and hence pressure gradient effects on this layer are larger than on the A-layer. 2) Kruka and Eskinazi have reported that even in the case of plane wall-jets in zero pressure gradient, no universal behavior for all injection rates is observed for $(U_m - U_{min})$ and $y_{1/2}$ in terms of either x_e/t_c or x/t_c. They found that universal relations for the growth of these parameters could be obtained only if a reduced longitudinal coordinate was used that took into account the relative displacement between the fluid particles in the B-layer and those in the freestream. Obviously such a coordinate cannot be evolved in a flow subjected to a pressure gradient since the freestream velocity itself will be changing with x.

Hence, for the B-layer, for small values of β, the wall-jet parameters can be expected to be functions of x/t_c, U_j/U_o and R/R_c. Again, as in the case of the A-layer variables, correlation with plane wall-jets is possible only if the effect of geometry can be accounted for through the use of stretched parameters. Study of a limited amount of data indicate that the relevant stretched parameters for the B-layer are

$$(U_m - U_{min})/U_j \cdot R_c/R \quad \text{and} \quad (y_{1/2}/t_c) \cdot R_c/R$$

No physical justification can, however, be given for the choice of these variables. In Fig. 9, the data for runs 500 and 600 are shown along with the data for plane wall-jets from Ref. 4. The value of U_j/U_o in all these cases was approximately 2. The average value of β for run 600 was 1 while that for run 500 was 2. It can be seen that the development of both $(U_m - U_{min})$ and $y_{1/2}$ for run 600 correlate very well with the plane wall-jet data. The B-layer behavior at a slightly higher value of $\beta = 2$ is seen to be different from that of the plane wall-jet even though U_j/U_o is the same in both cases. Unfortunately, although plane wall-jet data at higher values of U_j/U_o are available, values of $U_j/U_o > 2.38$ were not studied in the present investigation since they were outside the range of interest to the diffuser designer. Data for lower values of U_j/U_o cannot be used for comparison since β will be large in such cases. Further data are, therefore, needed to confirm whether the proposed stretched parameters are satisfactory at all values of U_j/U_o.

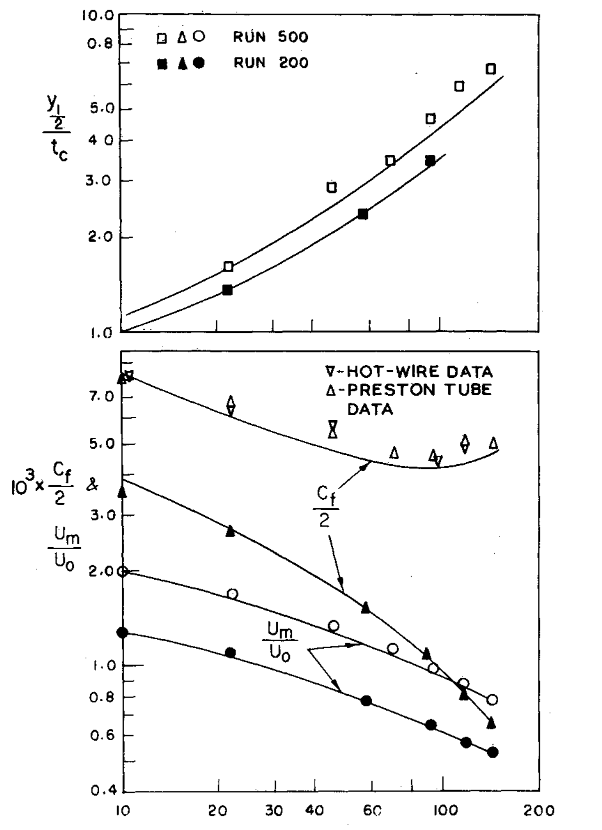

Fig. 10 Wall-jet development: comparison of predictions with experiment. Data for $\theta = 5°$.

3. The Prediction of the Wall-Jet Growth

The discussion so far has clearly demonstrated that even for moderately large values of β, it becomes difficult to arrive at algebraic relations expressing the development of the wall-jet parameters. It has been found[1,2] that for optimum efficiency of diffuser operation, the value of U_j/U_o is to be near the lowest values of U_j/U_o studied in the present case. Under these conditions, the effect of pressure gradient may be quite large and the flow may be far from equilibrium. In order to predict such flows, it will be necessary to solve the basic differential equations.

In the present case the boundary-layer partial differential equations for momentum and continuity were solved by the finite-difference method of Patankar and Spalding.[14] The relevant equations for the present problem are

continuity

$$\partial(Ur)/\partial x+\partial(Vr)/\partial y = 0 \tag{11}$$

momentum

$$\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial y} = -\frac{dP}{dx}+\frac{\partial}{\partial y}(\tau-\rho\overline{uv}) \tag{12}$$

Prandtl's mixing length hypothesis in the following form was used to evaluate the shear stress term

$$(\tau-\rho\overline{uv}) = \left\{\rho l_m^{\,2}\left|\frac{\partial U}{\partial y}\right|+\rho v\right\}\frac{\partial U}{\partial y} \tag{13}$$

where l_m is the mixing length.

The following two-layer model was assumed for the distribution of l_m in the cross stream direction

$$\begin{aligned} l_m &= \kappa y \quad \text{for} \quad 0 < y < \lambda y_1/\kappa \\ l_m &= \lambda y_1 \quad \text{for} \quad y \geqq \lambda y_1/\kappa \end{aligned} \tag{14}$$

Here y_1 is a characteristic width of the wall-jet. In the present calculations it was taken as the distance from the wall to the farthest point where the velocity differed by 1% from the free-stream velocity. κ and λ are constants usually taken as about 0.4 and 0.1, respectively, in boundary-layer calculations (e.g., Ref. 14) but left to be adjusted in the present calculations. The pressure gradient was supplied from the measured distribution of U_e along the diffuser. The x-axis was chosen along the diffuser wall and the y-axis along the inward normal to the wall. The solution was started by supplying the measured velocity profile at $x/t_c = 10$ as the initial value.

Solutions were obtained for the run 200 by varying κ and λ systematically one at a time. The pair of values of κ and λ that gave the best agreement with experiments in respect of U_m and $y_{1/2}$ was selected. As can be seen from Fig. 10 it was possible to arrive at such values for κ and λ that would make the predictions nearly coincide with experimental data for run 200. For the rest of the runs, the same values of the constants were used. The following values were chosen for κ and λ:

$$\kappa = 0.435, \qquad \lambda = 0.125$$

It is to be stated here that the predictions were fairly sensitive to the values of these constants. The percentage change in the predicted values were roughly of the same magnitude as the percentage change in the value of the constants. The predictions of u_* and U_m were sensitive mainly to the value of κ while the prediction of $y_{1/2}$ was sensitive to the choice of λ. Figures 10–12 show the comparison of the predicted development of some of the wall-jet parameters with the measurements. It can be seen that the predictions are generally satisfactory. In Fig. 13 are shown the calculated velocity distributions for a few typical runs. These are compared with the experimental data. A study of the figure brings out that while the prediction of detailed velocity profiles is fairly satisfactory, it is not as good as that of the gross parameters. The predictions are especially poor near regions of velocity maximum and minimum. This is to be expected since the mixing length hypothesis is anomalous in such regions. It is in fact very encouraging to find that satisfactory predictions can be obtained from such a simple physical model. The present procedure, thus, appears to be adequate for many engineering applications.

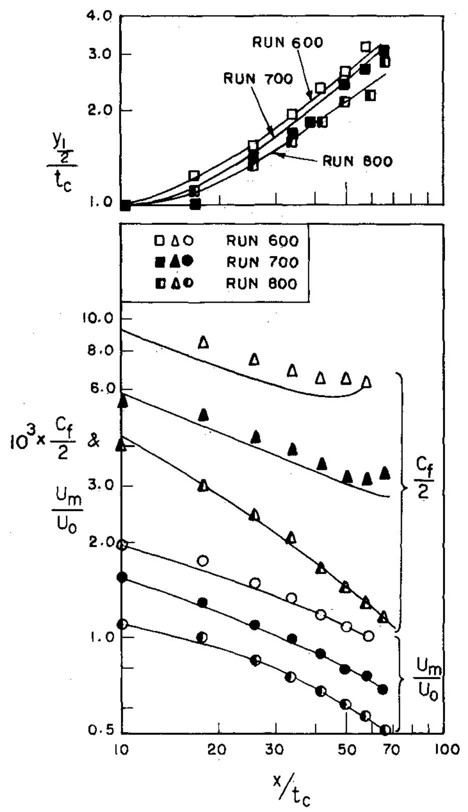

Fig. 11 Wall-jet development: comparison of predictions with experiment. Data for $\theta = 10°$.

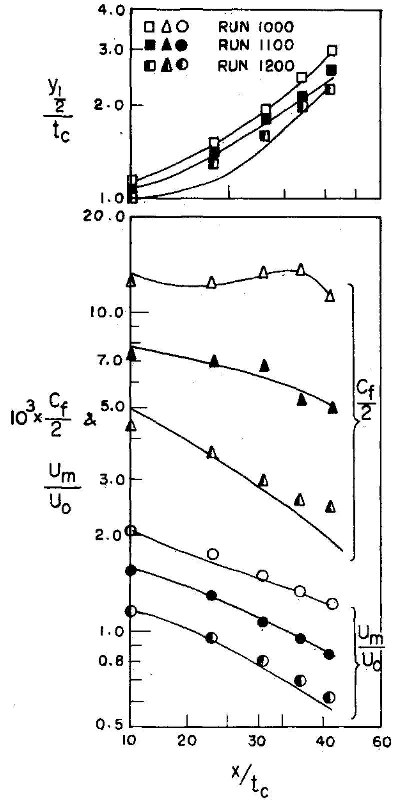

Fig. 12 Wall-jet development: comparison of predictions with experiment. Data for $\theta = 15°$.

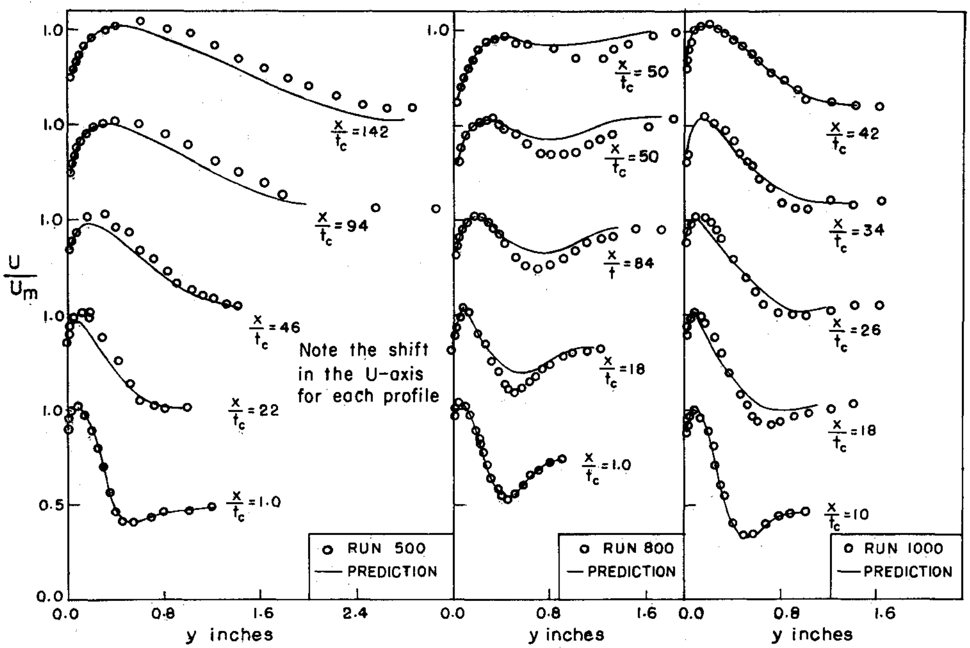

Fig. 13 Comparison of predicted and measured velocity profiles in a few typical cases.

4. Conclusions

The present investigation leads to the following conclusions.

1) Wall-jets in conical diffusers exhibit a universal logarithmic velocity distribution in the fully turbulent part of the wall region, provided the pressure gradient parameter β is less than 1. The logarithmic distribution can be represented by the equation $U^+ = 5.5 \log y^+ + 11.2$.

2) The growth of wall-jets in conical diffusers can be correlated with that of plane wall-jets in zero-pressure gradient by using suitably stretched parameters that account merely for the effect of diffuser geometry provided the pressure gradient parameter β is sufficiently small. The restriction on the upper limit of β is more severe for the B-layer than for the A-layer.

3) The wall-jet development in conical diffusers can be predicted satisfactorily using a simple two-layer mixing length model of turbulence.

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